Semi-Analytic Solution of the Horizontal Well Intersected by Multiple Finite Conductivity Fractures

by

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I. INTRODUCTION

The fracturing of horizontal wells has gained wide acceptance as a viable completion option to maximize the return on investment (i.e. increased well productivity). This is especially true in the case of tight gas formations. Compared to an openhole horizontal well, this type of completion provides a greater area of penetration and a better flow efficiency. It also provides more evenly...
pressure drop in the reservoir compared to a vertical well intersecting a vertical fracture, which enhances productivity.

Depending on the well orientation with respect to the minimum horizontal stress and length of the perforated interval, either a transverse or a longitudinal fracture may be created. If the horizontal well is drilled parallel to the minimum horizontal stress, it is expected that the created fractures will be perpendicular to the horizontal well, i.e. transverse fractures will be created. If the horizontal well is drilled perpendicular to the minimum horizontal stress, the created fracture will be longitudinal (axial). These two cases represent the two limiting and recommended cases. A more complicated situation is likely to occur if the well is at a different angle to the minimum horizontal stress.

This report presents the production prediction of a horizontal well intersected by perpendicular multiple fractures. Meanwhile the performance of a horizontal well intersected by a longitudinal fracture is almost identical to the performance of a vertical well intersecting a vertical fracture (Soliman et al., 1990) for the same fracture length and conductivity. Thus, production prediction of a horizontal well intersected by a longitudinal fracture can be simulated using a vertical well intersecting a vertical fracture model and it will not be discussed in this report.

In 1990, Soliman et al. examined the fracturing aspects of horizontal wells. They proposed an analytical model for a horizontal well intersecting multiple perpendicular fractures. For transverse circular fractures they obtained a Laplace transformed solution valid for the period when the flow in the reservoir can be treated as linear (towards the fracture plane). This model is valid for all time if the fracture half-length and drainage radius is equal. Otherwise, the time range validity of this solution is dependent on the ratio of fracture half-length to the drainage radius, and on the reservoir and fluid properties. It was also considered that the flow into the well occurred only through the well-fracture connection. One of their conclusions was that it is expected to observe a higher pressure drop than is usually observed in a vertical well intercepting a vertical fracture with similar conductivity. This is because of the convergence of streamlines inside the fracture toward the wellbore. Consequently, a higher \( F_{cD} \) may be necessary for designing this type of well completion. Conlin et al. (1990) examined the performance of six multiple fractured horizontal wells that were drilled in the Dan Field (Danish North Sea).

A numerical reservoir simulator was used to analyze performance and to forecast the production of the wells. It was concluded that multiple fractured horizontal wells are more cost effective than conventional deviated wells.

References (Raghavan et al., 1993, Roberts et al., 1991, and Hegre et al., 1994) proposed the equivalent radius concept for computing performance of multiple fractured horizontal wells. For these solutions to be valid, radial flow MUST be achieved. Therefore, these solutions are not suitable for a low permeability reservoir since the radial flow is very long to achieve.

Larsen et al. (1991) and Raghavan et al. (1994) proposed rigorous analytic solutions of the multiple fractured horizontal well. It was shown that the multiple fractured horizontal well responses could be correlated with a single fracture response at early time (before interference) and at late time (radial flow) (Raghavan et al., 1994). Methods were also provided to analyze pressure transient response of the multiple fractured horizontal wells for a number of possible flow periods (Larsen et al., 1990). Depending on the fracture length and the fracture orientation (transverse or longitudinal), multiple fractured horizontal wells may exhibit fracture radial, radial-linear (or bilinear), formation linear and pseudoradial flow prior to the onset of boundary effects. Despite their rigorousness, these solutions require a lengthy computation. Hence they are not very attractive when used in combination with a more complex problem, which may include multilayer, perforation and gravel pack, and flow in the wellbore.

Soliman et al. (1990) examined the effect of some parameters on the performance of multiple fractured horizontal wells. It concluded that the optimum number of fractures depends on reservoir and fluid properties as well as reservoir area. Also, it was observed that the solution for a vertical fracture intersecting a vertical well closely approximates the performance of a longitudinal fracture. Moreover, transverse fractures provide better coverage of the reservoir than longitudinal fractures, and thus they are more efficient in producing tight gas formation.

Some others (Hegre, 1996 and Escobar et al., 2001) performed a numerical modeling (finite difference model) with a more complex case (which includes arbitrary fracture orientation). Even though we can relax the assumptions using the finite difference model, the choice of a grid size and a grid pattern (i.e., cartesian, logarithmic, and unstructured grids) will likely
be case dependent (e.g. different dimensionless fracture conductivity may require different set of grid model). This may not be convenient for an inexperienced user or for a user who needs quick results.

The objective of this paper is to develop an approximate analytical (or semi analytical) solution for a multiple fractured horizontal well that is computationally efficient and accurate.

II. MODEL AND ASSUMPTIONS

The multiple fractured horizontal well is shown in Fig. 1. The fracture spacing, \( df \); is constant, and it may be different from the fracture length, \( xf \). It is also considered that the fracture length is the same for all fractures, and the horizontal well is cased and the flow into the wellbore only occurs through the well-fracture connection. The fluid production by individual fracture is not necessarily the same and it may vary with time (even if constant well production is imposed). The pressure inside the horizontal well is assumed uniform (or negligible pressure drop inside the horizontal well). The fracture height is equal to the net pay. The horizontal well intersects the middle of fractures. Because the wellbore radius is usually much smaller than the fracture height, the flow convergence toward the wellbore inside the fracture (Fig. 2) is also considered in the model.

The other assumptions are as follows:
- There is no flow across the upper and lower boundaries.
- The reservoir is horizontal and homogeneous with constant thickness.
- The reservoir is either laterally infinite or finite (with no-flow or constant pressure outer boundary).
- The pressure is uniform initially in the reservoir.
- The well production rate is constant.
- The fluid is slightly compressible (with constant compressibility) and has constant viscosity.
- The fluid flow obeys Darcy’s Law.
- The gravity effect is ignored.
- The system is isothermal

The convolution equation for the system is: [6]

\[
P_D(x_D, y_D, t_D) = \frac{c_s k h (p_i - p(x, y, t))}{q_{ref} \mu B} \quad \text{(4)}
\]

\[
t_D = \frac{c_s k t}{\phi \mu c_i L^2} \quad \text{...........................................(5)}
\]

\[
q_D = q(t) \quad \text{...........................................(6)}
\]

Applying Laplace transformation to Eqs. 1, 2 and 3 with respect to time variable, we obtain

\[
p_D(x, y, y_{D0}, u) = \sum_{i=1}^N u q_{Di} \bar{p}_{Di}([x_{D0}, [x_{Di}, y_{Di}, u]) \quad \text{(7)}
\]

\[
\bar{p}_{Di}(u) = \bar{p}_{Di}(u) \quad \text{...........................................(8)}
\]

\[
\bar{p}_{iD}(u) = \bar{p}_{iD}(u) \quad \text{...........................................(9)}
\]

Evaluating Eq. 7 and Eq. 9 at the fracture location \([x_k,y_k]\) we have

\[
p_D(x_{Di}, y_{Di}, u) = \sum_{i=1}^N u q_{Di} \bar{p}_{Di}([x_{Di}, y_{Di}, u])
\]

\[
\bar{p}_{Di}(u) = \bar{p}_{Di}(u) \quad \text{...........................................(10)}
\]

\[
\bar{p}_{iD}(u) = \bar{p}_{iD}(u) \quad \text{...........................................(11)}
\]

Writing Eqs. 10 and 11 for all fracture locations \((k=1,2,3,...,N)\) subject to Eq. 8, we have

\[
\begin{bmatrix}
\bar{p}_{D1} \\
\bar{p}_{D2} \\
\vdots \\
\bar{p}_{DN}
\end{bmatrix}
= \begin{bmatrix}
\bar{p}_{D1} \\
\bar{p}_{D2} \\
\vdots \\
\bar{p}_{DN}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\bar{q}_{D1} \\
\bar{q}_{D2} \\
\vdots \\
\bar{q}_{DN} \\
\bar{q}_{DW}
\end{bmatrix}
\quad \text{.............................................(12)}
\]

For constant well production rate, \(q_w=q_{ref}\), then we can write Eq. 12 as:

\[
\begin{bmatrix}
\bar{p}_{D1} \\
\bar{p}_{D2} \\
\vdots \\
\bar{p}_{DN}
\end{bmatrix}
= \begin{bmatrix}
\bar{p}_{D1} \\
\bar{p}_{D2} \\
\vdots \\
\bar{p}_{DN}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\bar{q}_{D1} \\
\bar{q}_{D2} \\
\vdots \\
\bar{q}_{DN} \\
\bar{q}_{DW}
\end{bmatrix}
\quad \text{.............................................(13)}
\]
Eq. 13 can be easily solved to obtain the flow rate from each fracture and the pressure in the wellbore. The Stehfest algorithm can be used to inverse the results from the Laplace domain into the real domain. Note that in Eq. 12 or 13, the pressure distribution due to a fracture is computed as follows. The elements of the matrix in the main diagonal (top left to bottom right) are computed using the finite conductivity fracture solutions. Otherwise, the uniform flux fracture solution is used. The fracture solutions may be obtained from reference (Marhaendrajana et al., 2001).

The effect of flow convergence may be computed using the equation proposed by Soliman (1990) and Schulte (1986). The solution presented in reference [1] is:

\[
\bar{p}_{wD} (u) = \frac{K_0 (r_{wD}) L}{u F_{D} L K_0 (r_{wD}) r_{nD}} \quad \text{..................(14)}
\]

\[
L = \left \lfloor \frac{2 \sqrt{u} + u}{F_{D} L} \right \rfloor \quad \text{.................................(15)}
\]

\[
r_{nD} = \frac{r_w}{x_f} \quad \text{...........................................(16)}
\]

Eq. 14 includes the effect of convergence flow from fracture into the wellbore. This solution assumes fracture penetration ratio equal to 1, so that the flow from reservoir is always linear. Consequently, for a fracture penetration ratio less than one, this solution is only valid for early time, before the pseudoradial flow is achieved. Schulte (1986) shows that the effect of flow convergence vanishes once the flow from reservoir becomes the prevailing factor, therefore, the effect of convergence flow in the fracture does not depend on the type of flow from reservoir. This observation leads us to obtain the pressure drop due to convergence flow from Eq. 14 using desuperposition.

When the well radius is set equal to the half of the fracture height, then there is no flow convergence from the fracture into the wellbore. At this condition, Eq. 14 becomes:

\[
\bar{p}_{wD} (u) = \frac{K_0 (h_{fD}) L}{u F_{D} L K_0 (h_{fD}) h_{fD}} \quad \text{..................(17)}
\]

\[
h_{fD} = \frac{h_f}{2} \quad \text{...........................................(18)}
\]

where:

\[
h_f \quad \text{: fracture height}
\]

The pressure drop due to the flow convergence from the fracture into the wellbore is then obtained by subtracting Eq. 17 from Eq. 14, that is:

\[
\bar{p}_{wD, convergence} (u) = \frac{1}{u F_{D} L} \left( \frac{K_0 (r_{wD}) L}{K_0 (r_{wD}) r_{nD}} - \frac{K_0 (h_{fD}) L}{K_0 (h_{fD}) h_{fD}} \right) \quad \text{..................(19)}
\]

Eq. 19 is then added to the fracture solution.

### III. CASE OF CONSTANT BOTTOM HOLE FLOWING PRESSURE

The dimensionless rate solution for the constant bottom hole flowing pressure can be obtained directly using the convolution relation in the Laplace domain

\[
q_{wD} (u) = \frac{1}{u \bar{p}_{wD} (u)} \quad \text{.................................(20)}
\]

### IV. VALIDATION EXAMPLES

In this section, we validate the results from Eq. 13 with a numerical reservoir simulator. The data used for this purpose are shown in Table 1.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir thickness, h</td>
<td>100 ft</td>
</tr>
<tr>
<td>Fracture half-length, x_f</td>
<td>400 ft</td>
</tr>
<tr>
<td>Distance between fractures</td>
<td>400 ft</td>
</tr>
<tr>
<td>No. of fractures</td>
<td>3</td>
</tr>
<tr>
<td>Well radius, r_w</td>
<td>0.25 ft</td>
</tr>
<tr>
<td>Drainage area, A</td>
<td>91.8 acre</td>
</tr>
<tr>
<td>Reservoir permeability, k</td>
<td>5 mD</td>
</tr>
<tr>
<td>Reservoir porosity, ( \phi )</td>
<td>0.2</td>
</tr>
<tr>
<td>Viscosity, ( \mu )</td>
<td>0.8 cp</td>
</tr>
<tr>
<td>Fluid compressibility, c_f</td>
<td>6x10^{-6} psi^{-1}</td>
</tr>
<tr>
<td>Pore compressibility, c_r</td>
<td>3x10^{-6} psi^{-1}</td>
</tr>
<tr>
<td>Formation volume factor</td>
<td>1.1788 bbl/STB</td>
</tr>
</tbody>
</table>

This validation is conducted for dimensionless fracture conductivity of 1, 5, and 50. In the numerical simulator, this is achieved by modifying the permeability of the grid blocks that are part of the fracture system, and all other data are maintained the same.

The numerical model is generated using a grid shown in Fig. 3. Small grid blocks are used near the fracture and larger grid blocks are used away from the fracture system. This is to capture the space variation of flux from the reservoir into the fracture. Vertically, the net pay is also divided into smaller grid blocks. The purpose is to capture the flow convergence toward the horizontal wellbore (at the center).

Results from the two solutions are plotted in Figs. 4 and 5 for a constant production rate case
and for a constant wellbore pressure case, respectively. Difference between the two solutions can be seen for dimensionless fracture conductivity \( F_{CD} \) of 1, and above \( F_{CD} \) of 5 the two solutions agrees very well. It is suspected that different grid block sizes and model may need to be selected to better capture the variation of flux rate entering the far low \( F_{CD} \) fracture and flow convergence. A more detail sensitivity study (which is not the scope of this study) may be needed to have better understanding for modeling this well system using the numerical model.

V. SIMULATION EXAMPLES AND DISCUSSION

Figs. 6 through 16 show the effect of several parameters (number of fractures, distance between fractures, dimensionless fracture conductivity and drainage area) on the performance of a horizontal well intersected by multiple perpendicular fractures. Otherwise stated, all comparisons made in these figures are based on equal total fracture length. For example, two fractures with 100 ft half-length each is compared with a single fracture of 200 ft half-length, or is compared with 4 fractures of 50 ft half-length each, etc.

Figs. 6 and 7 show the effect of number of fractures on production performance of the multiple fractured horizontal well. A Constant wellbore pressure is used to generate well production data. In Fig. 6, performance of the conventional fractured vertical well is also plotted. All of these results are generated using total fracture length of 800 ft, drainage area of 230 acre and dimensionless fracture conductivity 0.2. For the multiple fractures, the distance between the two outmost fractures is held the same (i.e., the distance between fractures is smaller with more fractures). To make comparison easier, equal distance between fractures is used.

We see that as the number of fractures increases, the production increases (Fig. 6). This may be explained that more fractures provide more entrance for the fluid to flow into the wellbore. In Fig. 7, we plot the cumulative oil ratio versus the number of fractures for various producing time. The cumulative oil ratio is the ratio between cumulative oil production of the multiple fractured horizontal well and the cumulative production of the openhole vertical well with the same reservoir and fluid properties. As the time increases the optimum number of fracture decreases, since as time increases the well with more fractures depletes more than the well with less number of fractures. To evaluate an optimum number of fractures, time constraint need to be considered as a factor since any well with different number of fractures drains the reservoir with different rate. In addition, as time progresses the interference between the fractures will have more and more effect on production.

For an infinite reservoir, well with more fractures is superior in a much longer period of time (Fig. 8). For example, in the 230 acre reservoir case, fractures at \( t=10 \) days the well with six fractures gives more cumulative production than the well with 5 fractures, but at \( t=500 \) days the cumulative production from both wells are about the same. On the other hand, the infinite acting reservoir case shows that the well with six fractures still performs better than the well with 5 fractures at \( t=500 \) days. However, a similar trend is observed that as the time increases the optimum number of fractures decreases primarily due to the interference. At higher dimensionless fracture conductivity, the optimum number of fractures decreases, as seen in Figs. 9 to 11. When the dimensionless fracture conductivity is very high, the use of more fractures is less attractive (Fig. 11) (note that dimensionless fracture conductivity of 100 is used to generate data in this figure). A summary of the relationship between dimensionless fracture conductivity and the optimum number of fractures in these examples is given in Fig. 12. The data used in this figure is taken at \( t=30 \) days.

An interesting observation from Fig. 6 is that the performance of a vertical fractured well is greater than a multiple fractured horizontal well with 2 fractures. The explanation is that the fluid flow inside the fracture converges toward the horizontal wellbore and it creates more pressure drop. This flow convergence does not occur for an open hole and fractured vertical well. Nevertheless, multiple fractures create more access to the different regions of the reservoir, which enhances productivity. As this overcomes the effect of convergence flow (at sufficient number of fractures), the production of the multiple fractured horizontal well is greater than the vertical fractured well. In this example, the minimum number of fractures is 3, in order to be more attractive than the vertical fractured well.

This pressure drop due to convergence flow is dependent upon the horizontal wellbore radius and the fracture conductivity. The smaller the wellbore radius the greater pressure drop is. The effect of the fracture conductivity is similar to that of horizontal wellbore radius. The smaller
the fracture conductivity the greater the pressure drop is.

In Figs. 13 to 16 we examine the effect of the distance between fractures. These are conducted using two fractures with equal length and are simulated for different dimensionless fracture conductivity (0.2 and 100) and different drainage area (230 acre and infinite).

The greater the distance between fractures the greater the production is (Fig. 13), since the interference is less likely to occur. In Fig. 14, we see that the optimum distance between fractures increases with time (2x_f at t=10 days increases to 10x_f at t=100 days) then decreases (about 2x_f at t=1000 days). The reason is that as time increases the interference becomes more pronounced, so that a larger distance between fractures is needed. The behavior of being less attractive to have more fractures at late times is because of the boundary effect (i.e. reservoir produced by a well with more fractures depletes faster). The observation about the late time behavior in Fig. 14 is consistent with the one shown in Fig. 15 for infinite acting reservoir. In this case, the optimum distance between fractures always increases with time, since there is no boundary effect.

The high dimensionless fracture conductivity case (Fig. 16) shows a similar trend. Also, this suggests that dimensionless fracture conductivity does not significantly affect the optimum distance between fractures.

Based on the results of this study, the design of the multiple fractured well configurations (number of fractures, and distance between fractures) depends on the fracture and reservoir properties, and depends on the time constraint. Even though the effect of cost is minimized when comparing between two different multiple fractured well configurations (by using the same total fracture length), a comprehensive economic analysis should also be included in the real well screening process.

VI. CONCLUSIONS

From this work we conclude that:

1. We have developed a semi-analytic solution for a multiple fractured horizontal well. This solution is computationally more efficient (i.e. faster) than the existing solutions and it is accurate.
2. The optimum number of fractures is dependent upon the reservoir and fracture properties.
3. The minimum distance between two fractures should also be considered in designing the horizontal well completion with multiple fractures to avoid early interference.
4. For very high dimensionless conductivity fracture, the number of fractures has little effect on the performance of the multiple fractured horizontal well.

NOMENCLATURE

\[ \begin{align*} 
 p & = \text{pressure at any point in the reservoir} \\
 p_{FD} & = \text{pressure distribution due to finite conductivity fracture} \\
 q & = \text{average flow rate of the individual fracture} \\
 q_w & = \text{well rate} \\
 t & = \text{time} \\
 k & = \text{reservoir permeability} \\
 h & = \text{net pay thickness} \\
 B & = \text{formation volume factor} \\
 \mu & = \text{viscosity} \\
 \phi & = \text{porosity} \\
 c_1 & = \text{total compressibility} \\
 q_{ref} & = \text{reference rate} \\
 L & = \text{characteristic length} \\
 x,y & = \text{space location} \\
 i & = \text{i-th fracture (i=1,2,3...)} \\
 c_1,c_2 & = \text{unit conversion constants} \\
 u & = \text{Laplace parameter} \\
 K_0 & = \text{modified Bessel function, second kind, zero order} \\
 K_1 & = \text{modified Bessel function, second kind, first order} \\
 F_{cD} & = \text{dimensionless fracture conductivity} \\
 \eta_{FD} & = \text{dimensionless fracture storage} \\
 r_w & = \text{well radius} \\
 x_f & = \text{fracture half-length} 
 \end{align*} \]

REFERENCES


Figure 1. Multiple transverse fractures configuration.

Figure 2. Flow in the fractures converges into the wellbore.

Figure 3. Areal and vertical view of grid block model used in the numerical simulator.
Figure 4. Comparison between numerical and semi-analytic results (constant production rate case).

Figure 5. Comparison between numerical and semi-analytic results (constant bottom hole flowing pressure case).
Figure 6. Effect of number of fracture (cumulative production versus time).

Figure 7. Effect of number of fracture.
Figure 8. Effect of number of fractures for an infinite acting reservoir.

Figure 9. Effect of number of fractures with $F_cD=1$. 
Figure 10. Effect of number of fractures with $F_{cD} = 5$.

Figure 11. Effect of number of fractures with $F_{cD} = 100$. 
Figure 12. Optimum Number of Fractures versus Dimensionless Fracture Conductivity.

Figure 13. Effect of distance between fractures (cumulative production versus time).
Figure 14. Effect of distance between fractures.

Figure 15. Effect of distance between fractures for an infinite acting reservoir.
Figure 16. Effect of distance between fractures with high dimensionless fracture conductivity.